Two-Photon and Two-gluon Decays of 0^{++} and 2^{++} P-wave Heavy Quarkonium States*

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By neglecting the relative quark momenta in the propagator term, the two-photon and two-gluon decay amplitude of heavy quarkonia states can be written as a local heavy quark field operator matrix element which could be obtained from other processes or computed with QCD sum rules technique or lattice simulation, as shown in a recent work on $\eta_{c,b}$ two-photon decays. In this talk, I would like to discuss a similar calculation on P-wave $\chi_{c0,2}$ and $\chi_{b0,2}$ two-photon decays. We show that the effective Lagrangian for the two-photon decays of the P-wave $\chi_{c0,2}$ and $\chi_{b0,2}$ is given by the heavy quark energy-momentum tensor local operator and its trace, the $\bar{Q}Q$ scalar density. A simple expression for χ_{c0} two-photon and two-gluon decay rate in terms of the $f_{\chi_{c0}}$ decay constant, similar to that of η_c is obtained. From the existing QCD sum rules value for $f_{\chi_{c0}}$, we get $5\,\mathrm{keV}$ for the χ_{c0} two-photon width, somewhat larger than measurement.

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I. INTRODUCTION

First of all, I would like to dedicate this talk to the memory of Professor Giuseppe Nardulli, who, with great kindness and generosity has initiated the long and fruitful collaboration I have with the members of the Physics Department and INFN at the University of Bari.

In the non-relativisitic bound state calculation [1, 2], the two-photon and two-gluon decay rates for P-wave quarkonium states depend on the derivative of the spatial wave function at the origin which has to be extracted from potential models, unlike the two-photon decay rate of S-wave η_c and η_b quarkonia which can be predicted from the corresponding J/ψ and Υ leptonic widths using heavy quark spin symmetry(HQSS) [3], there is no similar prediction for the P-wave χ_c and χ_b states and all the existing theoretical values for the decay rates are based on potential model

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calculations [1, 4–14].

Since the matrix element of a heavy quark local operator between the vacumm and P-wave quarkonium state is also given, in bound state description, by the derivative of the spatial wave function at the origin, one could express the P-wave quarkonium two-photon and two-gluon decay amplitudes in terms of the matrix element of a local operator with the appropriate quantum number, like the heavy quark $\bar{Q}Q$ scalar density or axial vector current $\bar{Q}\gamma_{\mu}\gamma_5 Q$. We have thus an effective Lagrangian for the two-photon and two-gluon decays of P-wave quarkonia in terms of heavy quark field operator instead of the traditional bound state description in terms of the wave function. This effective Lagrangian can be derived in a simple manner by neglecting the relative quark momentum in the heavy quark propagator as in non-relativistic bound state calculation. In this talk, I would like to report on a recent work [15] using the effective Lagrangian approach to describe the two-photon and two gluon decays of P-wave heavy quarkonia state, similar to that for S-wave quarkonia [3] . This was stimulated by the recent new CLEO measurements [16, 17] of the two-photon decay rates of the charmonium P-wave 0^{++}, χ_{c0} and 2^{++} χ_{c2} states. We obtain an effective Lagrangian for P-wave quarkonium decays in terms of the heavy quark energy-momentum tensor and its trace and that the two-photon and two-gluon decay rates of $\chi_{c0,2}$ and $\chi_{b0,2}$ can be expressed in terms of the decay constants $f_{\chi_{c0}}$ and $f_{\chi_{b0}}$, similar to that for η_c and η_b, which are given, respectively, by f_{η_c} and f_{η_b}. Then a calculation of $f_{\chi_{c0}}$ and $f_{\chi_{b0}}$ by sum rules technique [18, 19] or lattice simulation [20, 21] would give us a prediction of the P-wave quarkonia decay rates. In fact, as shown below, $f_{\chi_{c0}}$ obtained in [18] implies a value of 5 keV for the χ_{c0} two-photon width, somewhat larger than measurement. In the following I will present only the main results, as more details are given in the published paper [15].

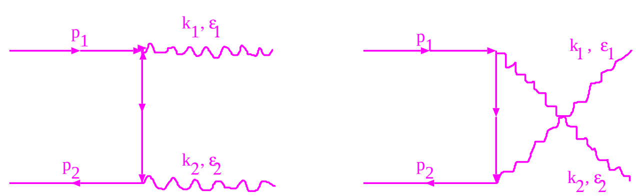

FIG. 1: Diagrams for $Q\bar{Q}$ annihilation to two photons.

II. EFFECTIVE LAGRANGIAN FOR $\chi_{c0,2} \to \gamma\gamma$ AND $\chi_{b0,2} \to \gamma\gamma$

By neglecting term containing the relative quark momenta q in the quark propagator[25] ($Q^2_{c,b}$ being the heavy quark charge), the P-wave part of the $c\bar{c} \to \gamma\gamma, gg$ and $b\bar{b} \to \gamma\gamma, gg$ amplitudes

represented by diagrams in Fig. 1 are

$$\mathcal{M}(Q\bar{Q} \to \gamma\gamma) = -e^2 Q_{c,b}^2 \frac{A_{\mu\nu}\bar{v}(p_2)T_{\mu\nu}u(p_1)}{[(k_1-k_2)^2/4 - m_Q^2]^2} \tag{1}$$

with $A_{\mu\nu}$ the photon part of the amplitude and the heavy quark part $T_{\mu\nu}$ given by

$$\begin{aligned} A_{\mu\nu} &= -2\epsilon_1 \cdot k_2 \epsilon_{2\mu} k_{1\nu} + 2\epsilon_1 \cdot \epsilon_2 k_{2\mu} k_{1\nu} \\ &\quad -2\epsilon_2 \cdot k_1 \epsilon_{1\mu} k_{2\nu} + (k_1 \cdot k_2)(\epsilon_{1\mu}\epsilon_{2\nu} + \epsilon_{2\mu}\epsilon_{1\nu}) \qquad (2) \\ T_{\mu\nu} &= (q_{1\mu} - q_{2\mu})\gamma_\nu \qquad (3) \end{aligned}$$

which can be obtained directly from the following effective Lagrangian for two-photon and two-gluon decay of P-wave heavy quarkonia states

$$\begin{aligned} \mathcal{L}_{\rm eff}(Q\bar{Q} \to \gamma\gamma) &= -ic_1 A_{\mu\nu}\bar{Q}(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\gamma_\nu Q \qquad (4) \\ c_1 &= -e^2 Q_{c,b}^2 [(k_1-k_2)^2/4 - m_Q^2]^{-2} \end{aligned}$$

With the matrix element of $\theta_{Q\mu\nu} = \bar{Q}(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\gamma_\nu Q$ between the vacuum and $\chi_{c0,2}$ or $\chi_{b0,2}$ given by ($Q^2 = M^2$)

$$\begin{aligned} <0|\theta_{Q\mu\nu}|\chi_0> &= T_0 M^2(-g_{\mu\nu} + Q_\mu Q_\nu/M^2), \\ <0|\theta_{Q\mu\nu}|\chi_2> &= -T_2 M^2 \epsilon_{\mu\nu}. \qquad (5) \end{aligned}$$

The two-photon decay amplitudes are then easily obtained :

$$\begin{aligned} \mathcal{M}(\chi_0 \to \gamma\gamma) &= -e^2 Q_{c,b}^2 \frac{T_0 A_0}{[M^2/4 + m_Q^2]^2} \qquad (6) \\ \mathcal{M}(\chi_2 \to \gamma\gamma) &= -e^2 Q_{c,b}^2 \frac{T_2 A_2}{[M^2/4 + m_Q^2]^2} \qquad (7) \end{aligned}$$

with $T_2 = \sqrt{3}T_0$ from HQSS and

$$\begin{aligned} A_0 &= (\frac{3}{2})M^2(M^2\epsilon_1 \cdot \epsilon_2 - 2\epsilon_1 \cdot k_2 \epsilon_2 \cdot k_1) \qquad (8) \\ A_2 &= M^2\epsilon_{\mu\nu}[M^2\epsilon_{1\mu}\epsilon_{2\nu} - 2(\epsilon_1 \cdot k_2 \epsilon_{2\mu} k_{1\nu} + \epsilon_2 \cdot k_1 \epsilon_{1\mu} k_{2\nu} \\ &\quad +\epsilon_1 \cdot \epsilon_2 k_{1\mu} k_{2\nu})] \qquad (9) \end{aligned}$$

For QCD sum rules calculation or lattice simulation, it is simpler to compute the trace of the energy-momentum tensor $\theta_{Q\mu\mu}$ given by $2m_Q\bar{Q}Q$. We have then

$$\bar{v}(p_2)T_{\mu\mu}u(p_1) = 2\, m_Q\, \bar{v}(p_2)u(p_1) \tag{10}$$

The problem of computing the two-photon or two-gluon decay amplitude of $\chi_{c0,2}$ and $\chi_{b0,2}$ states is reduced to computing the decays constants $f_{\chi_{c0}}$ and $f_{\chi_{b0}}$ defined as

$$< 0|\bar{Q}Q|\chi_0 >= m_{\chi_0} f_{\chi_0} \tag{11}$$

Thus T_0 is given directly in terms of f_{χ_0} without using the derivative of the P-wave spatial wave function at the origin.

$$T_0 = \frac{f_{\chi_0}}{3} \tag{12}$$

Thus by comparing the expression for χ_{c0} and η_c we could already have some estimate for the χ_{c0} two-photon and two-gluon decay rates. For $f_{\chi_{c0}}$ of $O(f_{\eta_c})$, one would expect $\Gamma_{\gamma\gamma}(\chi_{c0})$ to be in the range of a few keV.

The decay rates of $\chi_{c0,2}$, $\chi_{b0,2}$ states can now be obtained in terms of the decay constant f_{χ_0}. We have :

$$\Gamma_{\gamma\gamma}(\chi_{c0}) = \frac{4\pi Q_c^4 \alpha_{em}^2 M_{\chi_{c0}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c0}} + b)^4} \left[1 + B_0(\alpha_s/\pi)\right], \tag{13}$$

$$\Gamma_{\gamma\gamma}(\chi_{c2}) = \left(\frac{4}{15}\right) \frac{4\pi Q_c^4 \alpha_{em}^2 M_{\chi_{c2}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c2}} + b)^4} \left[1 + B_2(\alpha_s/\pi)\right] \tag{14}$$

where $B_0 = \pi^2/3 - 28/9$ and $B_2 = -16/3$ are NLO QCD radiative corrections [22–24]

This expression is similar to that for η_c :

$$\Gamma_{\gamma\gamma}(\eta_c) = \frac{4\pi Q_c^4 \alpha_{em}^2 M_{\eta_c} f_{\eta_c}^2}{(M_{\eta_c} + b)^2} \left[1 - \frac{\alpha_s}{\pi} \frac{(20 - \pi^2)}{3}\right] \tag{15}$$

The two-gluon decay rates are :

$$\Gamma_{gg}(\chi_{c0}) = \left(\frac{2}{9}\right) \frac{4\pi \alpha_s^2 M_{\chi_{c0}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c0}} + b)^4} [1 + C_0(\alpha_s/\pi)], \tag{16}$$

$$\Gamma_{gg}(\chi_{c2}) = \left(\frac{4}{15}\right) \left(\frac{2}{9}\right) \frac{4\pi \alpha_s^2 M_{\chi_{c2}}^3 f_{\chi_0}^2}{(M_{\chi_{c2}} + b)^4} [1 + C_2(\alpha_s/\pi)] \tag{17}$$

where $C_0 = 8.77$ and $C_2 = -4.827$ are NLO QCD radiative corrections. As with the two-photon decay rates, the expressions for two-gluon decay rates are similar to that for η_c:

$$\Gamma_{gg}(\eta_c) = \left(\frac{2}{9}\right) \frac{4\pi \alpha_s^2 M_{\eta_c} f_{\eta_c}^2}{(M_{\eta_c} + b)^2} \left[1 + 4.8\frac{\alpha_s}{\pi}\right] \tag{18}$$

In a bound state calculation, using the relativistic spin projection operator [25, 26] , f_{η_c} and f_{χ_0} are given by

$$f_{\eta_c} = \sqrt{\frac{3}{32\,\pi\, m_Q^3}}\, \mathcal{R}_0(0)\,(4\, m_Q)\ , \tag{19}$$

$$f_{\chi_0} = 12\sqrt{\frac{3}{(8\pi m_Q)}} \left(\frac{\mathcal{R}_1'(0)}{M}\right) \tag{20}$$

$$\tag{21}$$

which gives the decay amplitudes in agreement with the original calculation [1].

Comparing with f_{η_c}, we have

$$f_{\chi_{c0}} = 6\left(\frac{\mathcal{R}_1'(0)}{\mathcal{R}_0(0)M}\right) f_{\eta_c}. \tag{22}$$

which becomes comparable to f_{η_c}.

Thus by comparing the expression for χ_{c0} and η_c we could already have some estimate for the χ_{c0} two-photon and two-gluon decay rates. For $f_{\chi_{c0}}$ of $O(f_{\eta_c})$, one would expect $\Gamma_{\gamma\gamma}(\chi_{c0})$ to be in the range of a few keV. As shown in Table 1, the predicted two-photon width of χ_{c0} from the sum rules value $f_{\chi_{c0}} = 357\,\mathrm{MeV}$ [18] is however almost twice the CLEO value, but possibly with large theoretical uncertainties in sum rules calculation as to be expected, while a recent calculation [27] implies a larger decay rates for χ_{c0}. The measured ratio $\Gamma_{\gamma\gamma}(\chi_{c2})/\Gamma_{\gamma\gamma}(\chi_{c0})$ is then $\approx 0.24 \pm 0.09$, somewhat bigger than the predicted value of about 0.14 as shown in Table 1 together with the CLEO measurement of the decay rates [16] which gives $(2.53\pm0.37\pm0.26)\,\mathrm{keV}$ and $(0.60\pm0.06\pm0.06)\,\mathrm{keV}$ for χ_{c0} and χ_{c2} respectively.

Reference	$\Gamma_{\gamma\gamma}(\chi_{c0})(\mathrm{keV})$	$\Gamma_{\gamma\gamma}(\chi_{c2})(\mathrm{keV})$	$R = \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})}$
Barbieri[1]	3.5	0.93	0.27
Godfrey[4]	1.29	0.46	0.36
Barnes[5]	1.56	0.56	0.36
Gupta[7]	6.38	0.57	0.09
Münz[8]	1.39 ± 0.16	0.44 ± 0.14	$0.32^{+0.16}_{-0.12}$
Huang[9]	3.72 ± 1.10	0.49 ± 0.16	$0.13^{+0.11}_{-0.06}$
Ebert[10]	2.90	0.50	0.17
Schuler[11]	2.50	0.28	0.11
Crater[12]	$3.34 - 3.96$	$0.43 - 0.74$	$0.13 - 0.19$
Wang[13]	3.78	–	–
Laverty[14]	$1.99 - 2.10$	$0.30 - 0.73$	$0.14 - 0.37$
This work	5.00	0.70	0.14
Exp(CLEO)[16]	$2.53 \pm 0.37 \pm 0.26$	$0.60 \pm 0.06 \pm 0.06$	$0.24 \pm 0.04 \pm 0.03$
Exp(Average)[16]	$2.31 \pm 0.10 \pm 0.12$	$0.51 \pm 0.02 \pm 0.02$	$0.20 \pm 0.01 \pm 0.02$

TABLE I: Potential model predictions for $\chi_{c0,2}$ two-photon widths compared with this work.

The two-photon $\chi_{c0,2}, \chi'_{c0,2}$ branching ratios are independent of $f_{\chi_{c0}}$

$$\mathcal{B}(\chi_{c0}, \chi_{c'0} \to \gamma\gamma) = \frac{9}{2} Q_c^4 \frac{\alpha_{em}^2}{\alpha_s^2}\left(1 + (B_0 - C_0)\frac{\alpha_s}{\pi}\right) \tag{23}$$

$$\mathcal{B}(\chi_{c2}, \chi_{c'2} \to \gamma\gamma) = \frac{6}{5} Q_c^4 \frac{\alpha_{em}^2}{\alpha_s^2}\left(1 + (B_2 - C_2)\frac{\alpha_s}{\pi}\right) \tag{24}$$

with $B_0 = \pi^2/3 - 28/9, B_2 = -16/3, C_0 = 8.77, C_2 = -4.827$. Apart from QCD radiative correction factors, the expressions for branching ratios are very similar to that for η_c and η_c':

$$\mathcal{B}(\eta_c, \eta_{c'} \to \gamma\gamma) = \frac{9}{2} Q_c^4 \frac{\alpha_{em}^2}{\alpha_s^2}\left(1 - 8.2\,\frac{\alpha_s}{\pi}\right) \tag{25}$$

with α_s evaluated at the appropriate scale.

For $\alpha_s = 0.26$, $\mathcal{B}(\eta_c \to \gamma\gamma) = 3.6 \times 10^{-4}$ to be compared with the measured value of $(2.8 \pm 0.9) \times 10^{-4}$ [17], but this prediction is rather sensitive to α_s, for example, with $\alpha_s = 0.28$, one would get $\mathcal{B}(\eta_c \to \gamma\gamma) = 2.95 \times 10^{-4}$, in better agreement with the measured value of $(2.4^{+1.1}_{-0.9}) \times 10^{-4}$ and for $\chi_{c0,2}$, the predicted two-photon branching ratios would be 3.45×10^{-4} and 4.45×10^{-4} compared with the measured values of $(2.35 \pm 0.23) \times 10^{-4}$ and $(2.43 \pm 0.18) \times 10^{-4}$, for χ_{c0} and χ_{c2} respcetively. The predicted branching ratio for χ_{c2} is rather large and one would need $\alpha_s = 0.36$ to bring the predicted value closer to experiment.

Recently, the $Z(3930)$ state above $D\bar{D}$ threshold found by Belle [28] with mass $(3928 \pm 5 \pm 2)$ MeV and width $(29 \pm 10(\mathrm{stat}) \pm 2(\mathrm{sys}))$ MeV, consistent with χ_{c2}', seems to be confirmed by the observation of a similar state by BaBar [29], with mass $(3926.7 \pm 2.7 \pm 1.1)$ MeV and width $(21.3 \pm 6.8 \pm 3.6)$ MeV. Belle [28] gives $\Gamma_{\gamma\gamma}(\chi_{c2}') \times \mathcal{B}(D\bar{D}) = (0.18 \pm 0.05 \pm 0.03)$ keV while Babar [29] gives $\Gamma_{\gamma\gamma}(\chi_{c2}') \times \mathcal{B}(D\bar{D}) = (0.24 \pm 0.05 \pm 0.04)$ keV for this state. If taken to be the $2P$ excited state χ_{c2}' and assuming $\mathcal{B}(D\bar{D}) \approx 0.70 - 1$ [30–32], one would get $\Gamma_{\gamma\gamma}(\chi_{c2}') = (0.18 - 0.24 \pm 0.05 \pm 0.03)$ keV . This implies $f_{\chi_{c0}'} \simeq (195 - 225)$ MeV and $\Gamma_{gg}(\chi_{c0}')$ in the range $(5 - 10)$ MeV.

For $\chi_{b0,2}$ potential model calculations similar to that for $\chi_{c0,2}$, gives the two-photon width about 1/10 of that for η_b , which implies $f_{\chi_{b0}} = f_{\eta_b}/3$, smaller than Cornell potential [33] value $f_{\chi_{b0}} = 0.46\, f_{\eta_b}$.

III. REMARK ON THE η_c' TWO-PHOTON DECAYS

Since the predicted two-photon branching ratios for $\chi_{c0,2}, \chi_{c0,2}'$ and for η_c, η_c' are similar and independent of the decay constants, apart from QCD radiative corrections, as seen in Eq. (23-24) and Eq. (25), one expects a large two-photon decay rates for η_c', it would be relevant here to mention the problem of the $\eta_c' \to \gamma\gamma$ decay rate [3, 35]. The small value of $\Gamma_{\gamma\gamma}(\eta_c') = (1.3 \pm 0.6)$ keV given previously by CLEO [34] is obtained by assuming $\mathcal{B}(\eta_c' \to K_S K\pi) \approx \mathcal{B}(\eta_c \to K_S K\pi)$. However, with the recent BaBar measurement of the ratio [36]

$$\begin{aligned} R(\eta_c(2S)K^+/\eta_c K^+) &= \frac{\mathcal{B}(B^+ \to \eta_c(2S)K^+) \times \mathcal{B}(\eta_c(2S) \to K\bar{K}\pi)}{\mathcal{B}(B^+ \to \eta_c K^+) \times \mathcal{B}(\eta_c \to K\bar{K}\pi)} \\ &= 0.096^{+0.020}_{-0.019}(\mathrm{stat}) \pm 0.025(\mathrm{syst}) \end{aligned} \tag{26}$$

and the Belle measurement [37]

$$\mathcal{B}(B^+ \to \eta_c K^+) \times \mathcal{B}(\eta_c \to K\bar{K}\pi) = (6.88 \pm 0.77^{+0.55}_{-0.66}) \times 10^{-5} \tag{27}$$

BABAR obtains [36]

$$\mathcal{B}(\eta_c' \to K_S K\pi) = (1.9 \pm 0.4(\text{stat}) \pm 1.1(\text{syst}))\%. \tag{28}$$

as quoted by CLEO [38]. This new BABAR value for $\mathcal{B}(\eta_c' \to K_S K\pi)$ is considerably smaller than the corresponding value $\mathcal{B}(\eta_c \to K_S K\pi) = (7.0 \pm 1.2)\%$ [17] for η_c.

Thus with the BaBar result for $\mathcal{B}(\eta_c' \to K_S K\pi)$ and the CLEO measurement [34]

$$R(\eta_c'/\eta_c) = \frac{\Gamma_{\gamma\gamma}(\eta_c') \times \mathcal{B}(\eta_c' \to K_S K\pi)}{\Gamma_{\gamma\gamma}(\eta_c) \times \mathcal{B}(\eta_c \to K_S K\pi)} = 0.18 \pm 0.05 \pm 0.02 \tag{29}$$

one would get [38]

$$\Gamma(\eta_{c'} \to \gamma\gamma) = (4.8 \pm 3.7)\,\text{keV} \tag{30}$$

in agreement with the predicted value

$$\Gamma(\eta_{c'} \to \gamma\gamma) = (4.1 \pm 2.3)\,\text{keV} \tag{31}$$

while the assumption of near equality of the $K_S K\pi$ branching ratios for η_c and η_c'

$$\mathcal{B}(\eta_c' \to K_S K\pi) \approx \mathcal{B}(\eta_c \to K_S K\pi) \tag{32}$$

and the Belle ratio [39]

$$R(\eta_c' K/\eta_c K) = \frac{\mathcal{B}(B \to K\eta_c(2S) \times \mathcal{B}(\eta_c(2S) \to K_S K^- \pi^+)}{\mathcal{B}(B^0 \to K\eta_c) \times \mathcal{B}(\eta_c \to K_S K^- \pi^+)} = 0.38 \pm 0.12 \pm 0.05 \tag{33}$$

would lead to [38]

$$\Gamma_{\gamma\gamma}(\eta_c') = (1.3 \pm 0.6)\,\text{keV} \tag{34}$$

which is rather small compared with the predicted value given in Eq. (31) above.

IV. CONCLUSION

In conclusion, we have derived an effective Lagrangian for $\chi_{c0,2}$ and $\chi_{b0,2}$ two-photon and two-gluon in terms of the decay constants $f_{\chi_{c,b0}}$, similar to that for $\eta_{c,b}$ in terms of $f_{\eta_{c,b}}$.

Existing sum rules calculation, however produces a two-photon width about 5 keV, somewhat bigger than the CLEO measured value. It remains to be seen whether a better determination of $f_{\chi_{c0}}$ from lattice simulation or QCD sum rules calculation could bring the $\chi_{c0,2}$ two-photon decay rates closer to experiments or higher order QCD radiative corrections and large relativistic corrections are needed to explain the data.

The problem of two-photon width of η_c' would go away if more data could confirm the small BaBar value for $\mathcal{B}(\eta_c' \to K_S K\pi)$ compared with $\mathcal{B}(\eta_c \to K_S K\pi)$.

As relativistic corrections should be small for P-wave bottomia $\chi_{b0,2}$ states, two-photon and two-gluon decays could provide a test of QCD and a determination of α_s at the the m_b mass scale.

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